Apriorism and Scientific Cooperation in Hegel

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Hegel's commentators often attribute to his system some form of apriorism, the view that the system's content or its justification (or both) are independent of experience and empirical science. In this article, I argue that apriorism conflicts with Hegel's commitment to cooperation between the philosophical and empirical sciences, as outlined in §§1–18 of the 1830 *Encyclopaedia*. I do so by attributing two theses to Hegel: *scientific cooperation*—that knowledge arises through a process of conceptual transformation which requires an intellectual division of labour between the philosophical and empirical sciences; and *incompatibility*—that scientific cooperation entails a feedback loop between the philosophical and empirical sciences, rendering the concepts of Hegel's system intrinsically empirically revisable, and so not a priori. Although these two theses hold across all the philosophical sciences, I focus on their application in logic, as it is in logic where apriorist interpretations appear the most justified. Reimagining a scientifically cooperative Hegel not only supports naturalist readings of his system but also reframes the task of philosophical critique. Critique, on the scientific-cooperative reading I propose, aims to exposit the insights, discoveries, and theories of the empirical sciences, furthering their ends by ameliorating their conceptual apparatus, not to debunk them.

Keywords: Hegel, a priori, empirical science, conceptual transformation, critique

1 Introduction

Hegel's commentators often attribute to his system some form of apriorism, the view that the system's content or its justification (or both) are independent of experience and empirical science.[1] In this article, I argue that apriorism conflicts with Hegel's commitment to cooperation between the philosophical and empirical sciences, as outlined in §§1–18 of the 1830 *Encyclopaedia*. Because Hegel nowhere defends apriorism explicitly, this conflict provides indirect grounds for rejecting aprioristic readings of Hegel's system, including logic.

[1] Experience in this article is understood in a sufficiently *narrow* sense, restricted to sensation, perception, and their subjective modifications in memory, imagination, etc. This narrow concept of experience can be distinguished from a *broad* one, which would encompass the experience of consciousness and thinking as such. In debates over apriorism, it is common to assume that the relevant concept of experience must be sufficiently narrow. This is because an overly expansive concept of experience, for example, one which would include the mental processes of reflecting on mathematical and logical inferences, would render apriorism trivially false. Hegel, of course, develops both broad and narrow concepts of experience; however, to avoid assuming what I intend to demonstrate, I focus only on the narrow concept. On this point, see BonJour (1998, 6–11); Dunphy (forthcoming, 210n46).

I will defend this view by attributing the following two theses to Hegel:

1. *Scientific cooperation*: Philosophical and empirical science do and must cooperate to produce genuine knowledge. Knowledge requires not only that philosophy takes up and modifies the results and categories of the empirical science, but equally that empirical science adjusts its research programs in light of philosophy's conceptual transformation. The relationship between the empirical and philosophical sciences is a two-way street.
2. *Incompatibility*: Scientific cooperation is incompatible with apriorism. Once we see that cooperation between the philosophical and empirical sciences produces a looping effect and that this circuit continuously integrates new experiential content and empirical-scientific results, it becomes evident that the conceptual architecture of Hegel's system entails its intrinsic empirical revisability, that even logic must in principle be revised on the basis of future empirical insight and discovery. This intrinsic empirical revisability contradicts apriorism, so Hegel's scientific cooperation provides *indirect* evidence that he abandons it.[2]

Hegel's scientific cooperativism, if I am correct in attributing it to him, has two noteworthy consequences for our understanding of his general philosophical project. First, it underscores his naturalism because it demands that philosophy be responsive to advances in the empirical sciences. As the sciences progress or change, so must philosophy, insofar as philosophy involves, but is not exhausted by, exhibiting the concepts of empirical science. This responsiveness is part of what it means for philosophy to be "*its own time comprehended in thoughts*" (PR 21/GW 14,1:15).[3] Second, Hegel's scientific cooperativism also modifies our understanding of the nature of philosophical critique, underscoring its *expository* function. Contrary to many apriorist readings of Hegel, which assert that Hegel's critique of the science's finite cognition is primarily denunciatory, on the cooperative view, philosophical critique does not debunk the categories and assumptions of the

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[2] Contemporary philosophers sometimes call this immunity to revision "incorrigibility" (Casullo 2013, 253). Although some philosophers dispute that apriority entails incorrigibility, this entailment is assumed by commentators in disputes over Hegel's apriorism.

[3] Abbreviations used: *EG* = Hegel, *Hegel's Philosophy of Mind*, trans. M. J. Inwood (Oxford: Oxford University Press); *EL* = Hegel, *Encyclopaedia of the Philosophical Sciences in Basic Outline, Part 1: The Science of Logic*, trans. Klaus Brinkmann and Daniel Dahlstrom (Cambridge: Cambridge University Press, 2010); *EN* = Hegel, *Hegel's Philosophy of Nature*, trans Michael Petry, 3 vols (London: George Allen and Unwin, 1970); *GW* = Hegel, *Gesammelte Werke*, ed. Rheinisch-Westfälische Akademie der Wissenschaften (Hamburg: Meiner, 1968–); *PR* = Hegel, *Elements of the Philosophy of Right*, trans. H. B. Nisbet (Cambridge: Cambridge University Press, 1991); *SL* = Hegel, *The Science of Logic*, trans. George Di Giovanni (Cambridge: Cambridge University Press, 2010); *W* = Hegel, *Werke in Zwanzig Bänden*, eds. Eva Moldenhauer and Karl Markus Michel, 20 vols (Frankfurt am Main: Suhrkamp, 1986).

other sciences, dissolving their claims to validity, but impels and improves them and their research programs by ameliorating their concepts.[4]

Debates concerning Hegel's apriorism have often focused on the *Naturphilosophie*, as this is the domain in which an a priori approach appears to be least defensible. But in this article, I take a different tack, focusing instead on the philosophical science of logic. I do so for two reasons. First, it is Hegel's logic that readers most commonly understand to be strictly a priori. Thus, if Hegel's logic proves not to be a priori in the relevant senses, then, *a fortiori*, so does the rest of the system. Second, the scientific cooperativism that I wish to attribute to Hegel is a view that concerns the relation between the philosophical and empirical sciences as a whole and thus generalizes across his system, obtaining for each of the philosophical sciences of logic, nature, and spirit. Understanding how this cooperation takes place in the domain of logic is more difficult than in the *Realphilosophie*, so scientific cooperation in logic most warrants our attention.

This article proceeds as follows. In §2, I justify my taking an indirect route to rejecting Hegel's apriorism by outlining the relative aporia that arises on the direct route (§2.1), reviewing common positions on apriorism and sorting them into three groups: insulative, reconstructive, and cooperative (§2.2). In §3, I defend the cooperative view by examining Hegel's philosophy of science in the 1830 *Encyclopaedia*'s introduction (§3.1), Hegel's analysis of an instance of breakdown in this cooperation in the *Encyclopaedia* logic (§3.2), and the difference between intrinsic and external empirical revisability (§3.3). In §4, I turn briefly to Hegel's critique of the infinitesimal in the *Science of Logic*'s quantity chapter to exemplify how Hegel takes his scientific cooperation to function in logic, showing the nature of philosophical critique to be expository rather than denunciatory or critical—or, if one likes, that it is critical only because of and to the extent that it is first expository. My aim throughout this article is not to present a conclusive argument against apriositic readings of Hegel, but to encourage further work on the non-aprioristic alternatives, such as the cooperative view I defend. Consequently, much of my discussion remains quite sketchy, and future research would do well to concretize these somewhat vague descriptions and anaylze further episdoes of scientific cooperation throughout Hegel's writings.

[4] My account of Hegel's view of philosophy as conceptual transformation is indebted to work presented at the Hegel and Empirical Judgment conference by Mark Alznauer, titlted "Hegel on Philosophy as Radical Conceptual Ameliorization," in June 2021.

2 Apriorism

Most commentators interpret Hegel's logic to be an a priori undertaking.[5] Take, for example, Robert Pippin's and Stephen Houlgate's recent monographs on Hegel's logic. Pippin characterizes Hegel as holding that "a priori knowledge of the world […] is possible—knowledge about that world, but achieved independently of empirical experience" and concludes "that the *Logic* is a work of a priori philosophy is hardly controversial" (2019, 4f). Houlgate corroborates Pippin's interpretation, writing, "I agree with Pippin [1989] that Hegel does, indeed, argue that certain a priori categories structure our thought and experience" and that the *Logic* "provides a logical 'reconstruction' of our ordinary categories […] by deriving [their] true structure […] immanently and purely a priori from the empty thought of pure, indeterminate being" (2006, 430, 99).[6] For Pippin and Houlgate, it goes almost without saying that Hegel's logic is a priori, despite their many other disagreements on how to interpret this project.

For some, but not all, commentators, Hegel's apriorism precipitates a negative evaluation of empirical science. Bowman (2013), for instance, infers from Hegel's apriorism that he holds a "dim view" of the empirical sciences (136, 156f). Longuenesse (2007) concludes on similar grounds that Hegel aims not at "modestly [following] the development of particular sciences" or "ground[ing] scientific discourses" but at "dissolv[ing] their claim to objective validity" (37f). Finally, Stone (2005) argues that, because of his apriorism in *Naturphilosophie*, "Hegel [elaborates] a *sui generis* theory of nature and thereby [articulates] a forceful critique of science with positive ecological implications" (30).[7] For these commentators, Hegel's apriorism entails that philosophy adopt a combative attitude towards empirical science, debunking its claims to truth.

Both of these interpretations are, I will suggest, incorrect, or at least questionable. There are good reasons for holding that Hegel neither treats logic as a priori nor disdains empirical science. Indeed, "*Naturphilosophie*," and, as I will argue, all of philosophy "so little despises experience that it rather presupposes it for its existence" (GW 24,1:490). But for the remainder of this section, I will

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[5] Exceptions to the apriorist tendency include Renault (2019); Rand (2007, 2017, 2021); Kreines (2015); Redding (2007); DeVries (1988). The most comprehensive treatment on Hegel's apriorism is Stone (2005). Orsini (2021) provides a helpful overview of the debate with respect to logic, as do Sala and Kabeshkin (2022) with respect to *Naturphilosophie*.

[6] Recently, Houlgate (2021) has reaffirmed that Hegel's logic is "a strictly a priori philosophy" (1:391n49, 1:102). See also Winfield (2012, 210–11); Pinkard, (1979, 417–18).

[7] Stone (2005) denies that her Hegel disdains empirical science (27, 89). However, I cannot see how Hegel's "forceful critique of science," on her interpretation, avoids this consequence.

focus on Hegel's rejection of apriorism in logic, returning to Hegel's related view of scientific cooperation in §3.

2.1 Direct and Indirect Routes to Rejecting Apriorism

We can distinguish between two ways of demonstrating Hegel's rejection (or acceptance) of apriorism. The first method may be called the *direct route*. It consists in examining the passages in which Hegel explicitly discusses the a priori and determining whether he affirms or rejects the view in each case. If some of these passages are ambiguous or contradictory, then either further passages can be consulted, or these passages can be shown to have a meaning that, despite first appearances, in fact supports one's preferred view. This process then continues until a consensus interpretation emerges. Ordinarily, we adopt the direct route to resolve interpretive disputes.

The second method is, by contrast, the *indirect route*. It consists in examining Hegel's other philosophical positions and evaluating whether they contradict the view in question or are compatible with it—in this case, apriorism. If any of these positions turn out to contradict apriorism, then this provides strong evidence that Hegel does not hold apriorism, given our background assumption that Hegel's particular views generally cohere with one another. Undoubtedly, the indirect route bears an increased degree of interpretive difficulty relative to the direct one. It is therefore appropriate only if the direct method fails to sufficiently prove or disprove Hegel's apriorism.

In the case of Hegel's apriorism, the direct method indeed leaves us in a state of relative interpretive indeterminacy, and we should adopt the indirect method to resolve it. To see why this is so, it is helpful first to briefly attempt the more commonly trodden direct route.

Consider the passages in the *Science of Logic* wherein Hegel discusses apriorism, of which there are approximately seven (SL 40ff, 71ff, 173ff, 519f, 524, 654f, 702). These passages indicate that Hegel rejects the a priori/a posteriori distinction for at least two reasons. First, in them, Hegel explicitly dismisses the distinction outright. He tells us, for instance, that the objective logic forgoes "the abstract form of the a priori against the a posteriori" (SL 42/GW 21:48f) and elsewhere calls the a priori a "vacuous expression" and "altogether all too vague" (SL 173/GW 21:199). Roughly paralleling his critique of any fixed separation between concept and intuition, Hegel's reasoning for this conclusion appears to be that each thought-determination examined in logic contains within it both moments of the a priori and a posteriori, and so our knowledge as constituted by these

thought-determinations cannot be classified as either a priori or a posteriori on pain of one-sidedness (EL §8A/GW 20:48f). Second, unlike most other basic concepts in the Western metaphysical tradition, Hegel does not actively integrate the a priori within his system. Nowhere does Hegel claim that his system is a priori; when he discusses the notion, he almost always does so negatively.[8] Taking the direct route, these passages provide good reason for interpreting Hegel as rejecting apriorism.

However, these grounds are unlikely to satisfy commentators such as Pippin, Houlgate, Bowman, and Longuenesse for at least three reasons. First, they may rejoin that these passages do not prove that Hegel abandons the a priori in logic as such but only Kant's abstract formulation of it. In their view, Hegel develops a *concrete* form of the a priori/a posteriori distinction, one which permits us to designate his logic as a priori, but in a sense different than Kant's. This reinterpretation explains why Hegel appears to dismiss apriority in the passages cited above.

Second, they would likely refer to their own passages that seemingly indicate Hegel accepts apriorism. These passages are of two kinds. On the one hand are passages in which Hegel refers to the a priori explicitly. These can be found in the 1830 *Encyclopaedia*.[9] On the other hand are passages that seem intelligible only if we ascribe to Hegel's logic some kind of apriorism but that do not themselves address the a priori explicitly or by name. In this regard, commentators often cite the loftiest claims Hegel makes for his logic: its being the science of pure thinking (SL 38/GW 21:45); its eternality, memorably encapsulated by Hegel's gloss that logic's content "is the *exposition of God as he is in his eternal essence before the creation of nature and of a finite spirit*" (SL 29/GW 21:33–34);[10] its affiliation with Kant's transcendental logic;[11] its giving itself, as the logical idea, its existence;[12] and its consisting solely in making explicit what is already contained in concepts.[13] For friends of apriorism, such features of Hegelian logic would be inexplicable if this logic were anything but a priori.

[8] See Rand (2021, 4–9; 2017, 388f).
[9] EL §12A/GW 20:53; EL §§40–41/GW 20:78–79.
[10] Cf. SL 463/GW 11:367; SL 674/GW 12:177; EL §235/GW 20:228.
[11] SL 40/GW 21:46–7; cf. SL 524/GW 12:27.
[12] SL 518/GW 12:20–21, cf. SL 707/GW 12:209.
[13] EL §88/GW 20:12.

Finally, these commentators might also complain of the general obscurity surrounding the idea of a science of logic that is not a priori. Without a forthcoming non-aprioristic alternative interpretation of Hegelian logic, we should retain the aprioristic reading.

However, these objections are, I believe, flawed and permit the following responses. First, the notion of a "more concrete" a priori is no less obscure than that of a non-aprioristic science of logic, and no defender of Hegel's apriorism has, to my knowledge, meaningfully distinguished Hegel's a priori from Kant's. Indeed, such interpreters usually do not develop their notion of the a priori in detail.[14] Yet the argumentative burden lies with them to provide such an account, given Hegel's apparent rejection of the a priori-a posteriori distinction. Second, the passages seemingly favorable to apriorism do not bear this out. Regarding those in which Hegel explicitly discusses the a priori, it is quite clear in these cases that Hegel is characterizing other philosophical positions, not his own. Regarding those that are merely suggestive of apriorism, they can be given an alternative interpretation that coheres with the non-apriorist view.[15] Finally, the obscurity of a science of logic that is not a priori largely proceeds from the anti-scientific prejudices of Hegel's commentators, not

[14] There are two exceptions to this tendency. The first is the assimilation of Hegel's a priori not to Kant's but to that of the pre-critical rationalists such as Wolff and Leibniz (Sala and Kabeshkin 2022). Certainly, this interpretation succeeds in differentiating Hegel's a prior from Kant's. However, it remains quite vague why this pre-critical version of the distinction would count as more 'concrete' than Kant's and unclear what motivates Hegel aims to return to Wolff on this score. Another exception to this tendency has been the attempt to reformulate the a priori as graduated. Posch (2011), for example, characterizes Hegel's *Naturphilosophie* as "*to a degree* a priori" and therefore as "limited" but not "inhibited" by its dependence on empirical science (181–83). Similarly, DeVries (1998) writes the a priori "is really a matter of degree" for Hegel, as it tracks levels of "empirical sensitivity," but also extends this view to logic, holding that "even logic is not absolutely a priori" (14n5, 33n1). Posch's and DeVries's reinterpretation of the a priori can, I believe, be understood as reviving the neo-Kantian idea of articulating a "relativized a priori," in which philosophy's a priori conceptual explication is theory-relative and so liable to revision (Friedman 2001; De Boer 2011; Herrmann 2014). However, this relativized, limited, or graduated a priori faces four serious challenges. First, nowhere does Hegel speak of the a priori as coming in degrees, a sort of talk he generally avoids, and so the view has no textual basis. Second, on the view of apriority assumed in much of the literature on Hegel's logic, it makes little sense to speak of logic as being *partially* a priori, as, on this view, it assumed that any experience-dependence suffices for the logic's categorization as a posteriori—in short, a relativized a priori is no a priori at all. Third, the graduated interpretation overlooks Hegel's aversion to the a priori/a posteriori distinction, which should be explained by our interpretation, not ignored. Fourth, the relativized a priori is often proposed as an interpretation of the *Realphilosophie*, but Hegel's rejection of apriorism also holds for logic. These four considerations suggest we are better off abandoning any commitment to the a priori in Hegel than reinterpreting it as relativized, limited, or graduated.

[15] For example, Orsini (2021) deflates Hegel's metaphor of logic being the exposition of God's thoughts before creation, rendering it compatible with a non-aprioristic interpretation of Hegelian logic (52–54).

from Hegel himself. Hegel's views on scientific cooperation help render intelligible the meaning and function of a science of logic that depends on empirical science.

Nonetheless, proponents of apriorism in Hegel are still likely unsatisfied. Partisans of either side can continue to refer to their preferred passages, and prospects for resolution between them remain slim. I therefore agree with Stone that the direct route—the consideration of only the explicit textual evidence—is unlikely to decide the issue because in it Hegel appears to adopt two incompatible positions regarding the a priori as he attempts to reconcile the rationalist and empiricist moments of scientific knowledge (Stone 2005, 1f, 5f, 8). To be sure, we do not aim to dissolve the interpretive dispute into aporia, but instead insist upon the necessity of systematic and philosophical-argumentative considerations in evaluating the issue due to there being insufficient direct textual evidence. Given this impasse, my strategy in the remainder of this essay will be to take the indirect route to putting apriorism into questoin, working backward from Hegel's scientific cooperativism to his rejection of apriorism (via incompatibility). However, before developing the cooperative view in detail, I will first map the interpretive terrain, contrasting the cooperative view with its alternatives.

2.2 Insulation, Reconstruction, and Cooperation

Interpretations of the relation between philosophy and empirical science in Hegel can be roughly classified into three groups: insulative, reconstructive, and cooperative. These interpretive standpoints designate a cluster of loosely connected views of the nature of the philosophical critique, the revisability of philosophical concepts, the value of empirical science, and the directional flow of conceptual content between the philosophical and empirical sciences. What follows is a brief sketch of this tripartite scheme; I will continue to fill in the cooperative view throughout the remainder of the article.

Construed aprioristically, philosophy adopts one of the first two standpoints. On the reconstructive view, philosophy surveys the results of the sciences and then rationally reconstructs them in form of the concept.[16] The relationship is 'bottom-up': the results of empirical science are unilaterally elevated into philosophy, and philosophy's relationship to finite cognition is primarily

[16] The reconstructive view corresponds to Stone's "weak a priorism." Its proponents include Hartmann (1966, 1972), Pippin (1989, 2019), Brandom (2005), Pinkard (1979, 1981), Sala & Kabeshkin (2022). Houlgate (1998, xiii–xiv) outlines a similar distinction between insulative and reconstructive readings of the science-philosophy relation in Hegel.

vindicatory, i.e., philosophy demonstrates its basic truth. The revisibility of these reconstructed philosophical concepts is variously understood as irrevisable (Brandom 2005, 156), as self-revising but empirically irrevisable (Pippin 2019, 25; 1989, 250), or as externally empirically revisible (Hartmann 1972, 122; 1966, 247; Pinkard 1979, 417; 1981, 453f). In the second case, philosophy's internal development may come to demand the revision of its concepts, but that this impetus arises only from within philosophy, not from empirical science, whereas in the third case empirical science may percipitate this revision by altering the given contents that philosophy ought to reconstruct. Although philosophy on the reconstructive view can be first oriented by the findings of empirical science, it is not a posteriori because the justification for its reconstructed conceptual scheme results entirely from its internal development of these findings, independent of experience. On the insulationist view, by contrast, philosophy begins by considering the forms of nature and spirit in themselves, without first examining their construal in empirical science.[17] It is 'top-down': only after completing its a priori conceptual scheme does philosophy then return to the empirical sciences, upon which it proffers critique—correcting or denouncing—any empirical-scientific theories or results that fail to correspond to its metaphysical scheme. This view usually conceives of philosophical concepts as either irrevisable or externally empirically revisible in a broadly Cartesian way, whereby revisions reflect only the correction of our methodological errors and are external to the concepts themselves.[18]

The cooperative view instead posits a bilateral and internal relation between empirical science and philosophy and, in turn, rejects apriorism.[19] It can be motivated by first noticing what is missing in both the reconstructive and insulative standpoints: namely, the possibility of a real collaboration between the philosophical and empirical sciences, one in which both the philosopher and the scientist would, in principle, be required to take a meaningful interest the other, concerning herself with the other's work, progress, and research programs. On the reconstructive view, it is evident why philosophers maintain an interest in the empirical sciences, but it is unclear why a scientific practitioners would concern herself with the results of this reconstruction, as philosophy, now resigned to merely rationally organizing the findings and theories of empirical science, only

[17] The insulationist or metaphysical view corresponds to Stone's "strong a priorism." Its proponents include Stone (2005), Bowman (2013), Longuenesse (2007), Houlgate (2006, 2021).
[18] A more detailed discussion of empirical revisability is postponed until §3.3 below.
[19] The suggestion that philosophy and empirical science in Hegel participate in a more or less cooperative enterprise can be found in Renault (2019); Rand (2017, 2021); Pinkard (2012); Buchdahl (1993); DeVries (1988).

demonstrates the grounds of her judgements antecedently taken as correct. Similarly, on the insulative view, the philosopher has no reason to concern herself with the advances of the empirical sciences, unless she wishes to rectify their mistakes, but this rectification is inessential for the progress of philosophical science as such. Real collaboration can be expected to arise only if a genuine need for it is felt by scientists and philosophers, which would not obtain on the insulative and reconstructive views. The cooperative view can instead be understood as modeling the relationship as an *intellectual division of labor*. For each branch of science, philosophy and empirical science undertake different parts of a single enterprise, modifying themselves in response to changes in the other. Success in this enterprise, then, requires real collaboration. This collaboration between science and philosophy is best accounted for in Hegel by generalizing a dictum found in his lectures on the philosophy of nature—that "physics and *Naturphilosophie* work hand in hand" (GW 24,1:490)—to all of the empirical and philosophical sciences.

The most notable consequence of taking a cooperative view on the relation between philosophy and empirical science is that it entails the empirical revisability of philosophy's system of concepts. That is, at least some of philosophy's concepts are subject to having their content, meaning, or validity altered because of advances or changes in the empirical sciences, and not merely because of philosophy's immanent self-development. As I explain in detail below, this is because philosophy involves conceptually transforming the abstract universals articulated by the empirical sciences. In this way, philosophy goes beyond merely according with empirical science because in its "*emergence*" [Entstehung] and "*formation*" [Bildung] it presupposes and is conditioned by empirical science (EN §246A/GW 20:236).

3 Scientific Cooperation

In expositing Hegel's views on scientific cooperation, I take as my point of departure his characterization of philosophical science as a conceptual modification of empirical science:

> To that extent, the relationship of the speculative to the other sciences is merely this, namely that the former does not simply set aside the empirical content of the latter, but instead recognizes [anerkennt] and uses it; that it likewise recognizes and utilizes as its own content the universal produced by these sciences, such as their laws, genera, etc.; and furthermore that it introduces into those categories others as well and validates them. In

> this respect, the difference between them concerns solely this modification [Veränderung] of the categories. (EL §9A/GW 20:49)

In this passage, Hegel construes philosophy as cooperating with empirical science in at least five ways. On the one hand, empirical science lends to philosophy its (1) empirical content and (2) its universals distilled thereof, which are "used" and "recognized" in philosophy. On the other hand, philosophy contributes to empirical science (3) additional concepts, such as freedom, spirit, and God (EL §8/GW 20:48), and (4) validates or justifies its use of these and other concepts. To these four cooperative moments, we may add a fifth, as Hegel tells us in the next sentence that logic "further builds and forms" the universals of the former metaphysics, which, if we generalize this to be a feature of philosophy's relation to empirical science as such, indicates that (5) philosophy modifies the meaning or conceptual content of empirical science's universals. Philosophy differs from empirical science, then, neither because it is independent of experience nor because it sets aside the latter's empirical data but because it embodies a distinct kind of conceptual transformation of the latter's categories.

Two further remarks should be made in this regard. First, this passage occurs in Hegel's *Einleitung* to the *Encyclopaedia* (§§1–18) and therefore pertains univocally to all the philosophical sciences. Although modern editions include this introduction as part of the *Encyclopaedia* logic, it lies outside of and prior to the particular philosophical sciences of logic, nature, and spirit—it is the introduction to the entire system of philosophy. This position explains why in the *Einleitung* Hegel sketches the relationship between the philosophical and empirical sciences as a whole. As Hegel reminds us later in the *Encyclopaedia*, in the *Einleitung* he addresses "the relation of philosophy to the empirical" in general (EN §248A/GW 20:236). So the views expressed in §9A, given their occurrence in the introduction to *all* of the philosophical sciences, apply in the same sense to each of the philosophical sciences, including logic, evident in the passage's next sentence addressing "speculative logic."

Second, the shift that I propose in our understanding of Hegel equally concerns the philosopher's attitude towards empirical science as it does her philosophical commitments. My contention is that this process of conceptual modification is expository and cooperative rather than merely corrective (as it is on the insulative reading) or vindicatory (as it is on the reconstructive one). Putting my thesis attitudinally, the empirical scientist should feel that philosophy receives her results

and exposits them in their truth rather than merely correcting or denouncing them (insulation) or exonerating them (reconstruction). Philosophy should show up to the scientist more like a helpful colleague than a dismissive critic or an obsequious underlaborer. This cooperative attitude requires that the philosopher adopt an expository attitude towards empirical science.[20] On the cooperative reading, then, the philosophical critique of empirical science comes to resemble something like Marx's critique of political economy: an exposition of a system of thought that, because it is a true conceptual exposition, is at the same time, and for only this reason, a critique of the system.[21]

My exposition of Hegel's scientific cooperativism in the *Encyclopaedia* proceeds in two subsections. First, I outline what we may call Hegel's "philosophy of science" in §§1–18 and philosophy's role therein as a form of conceptual modification (§3.1). Second, I examine a breakdown in this cooperation in the *Encyclopaedia* logic, wherein Hegel repudiates Kantian philosophy for failing to cooperate with empirical science in the right way (§3.3). Third, I argue that the right sort of cooperation entails that philosophy's concepts must be intrinsically, not merely externally, empirically revisable (§3.3). Then, in the article's concluding section, I exemplify Hegel's practice of concept modification in his critique of the infinitesimal calculus in the three long additions to the Quantity chapter in the *Science of Logic* (§4). Although the debate over Hegel's apriorism has primarily focused on his *Naturphilosophie*, these considerations show that Hegel consistently treats empirical science across all parts of his system, precisely in line with the *Einleitung*.[22]

3.1 Hegel's Philosophy of Science (EL §§1–18)

As I have already indicated, Hegel's philosophy of science in §§1–18 can be understood as a process of conceptual amelioration or conceptual transformation.[23] This process consists of the "modification" (*Veränderung*) of thinking (*Denken*) through its three stages or moments:

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[20] As Buchdahl (1993) puts it, philosophy aims at "constructive criticism" of the empirical sciences (62).
[21] See Marx's description of *Zur Kritik der politischen Ökonomie*: "The work […] is a critique of economic categories, or, if you like, the system of bourgeois economy critically exposited. It is at the same time an exposition of the system and, through the exposition, a critique of it" (Marx and Engels 2010, 270–71).
[22] For example, Hegel's appropriation of political economy in the doctrine of objective spirit should be analyzed as an instance of this kind of conceptual transformation (EL §7A/GW 20:47). Cf. Mooren and Rojek (2015, 89n43); Dunphy (2024, 205).
[23] Another important text for Hegel's scientific cooperation is the *Encyclopedia* logic's *Vorbegriff* (§§19–83), especially the "Second Position of Thought towards Objectivity" (§§37–60). I omit consideration of this text for reasons of space. I also set aside the important changes between the 1817 and 1827 editions, which tend towards a more empirical standpoint (Renault 2019).

representations (*Vorstellungen*), thoughts (*Gedanken*), and concepts (*Begriffe*).[24] These forms of thinking constitute a progressive series, moving from representations arising from sense-experience to fully mediated and self-developed concepts. Each also corresponds to a distinct "manner of cognizing" (*Erkenntnißweise*): sense-perception, empirical science, and philosophical science (EL §4/GW 20:43; EL §10/GW 20:50). Thus only the last stage, that which transforms thoughts into concepts, is distinctly philosophical. To better understand this process, we can first turn to Hegel's account of thinking in this context.[25]

In the *Einleitung*, thinking has two senses for Hegel, one generic and the other philosophical. Generically, thinking constitutes the essence of all distinctly human activity. Thus Hegel writes, "it is through thinking that human beings distinguish themselves from the *animals*" and "everything human is human as a result of and only as a result of thinking" (EL §2/GW 20:40). This generic thinking is "at work in everything human and which, indeed, is responsible for the humanity of all that is human" (ibid.). But, on the other hand, thinking also characterizes specific philosophy's mode of activity as "*thinking examination*" (*denkende Betrachtung*) of objects or "conceptual cognition" (*begreifendes Erkennen*) which differs from the generic sort of thinking by being explicit for thought in the form of concepts (cf. EG §465Z/W 10:284). However, for Hegel these two senses of thinking are not independent mental processes or mental substrata because "*in itself* there is only *one* thinking" (ibid.). Instead, they are distinguished by being different "*forms*" of thinking (ibid.), the diverse ways

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[24] Hegel's philosophy of science is relatively unexplored in the literature. The most comprehensive study is Mooren and Rojek (2015); see also Schick (2022); Renault (2019, 34–38, 42–43); DeVries (1988, xi–xii, 13–17, 28–46); Halbig (2002, 161–67). My exposition diverges from these existing treatments of the topic for two reasons: first, because I argue for a cooperative reading between the empirical and philosophical sciences; second, because I focus on the consequences of Hegel's philosophy of science for the apriority of his system, which remains only a second concern in these existing treatments.

[25] Hegel's discussion of the transformation of representations into thoughts and concepts in the *Einleitung* mirrors in many ways the later development of theoretical spirit in the philosophy of spirit's chapter on psychology (EG §§445–68). This development also proceeds from representations into thoughts and from thoughts into concepts. However, there are good reasons for doubting that these are *identical* processes for Hegel. Most notably, in the psychology Hegel clearly has in view the mental formation of a single human person, wherein the moments of theoretical spirit exhibit a non-linear or parallel structure, each moment being simultaneous with the others. By contrast, in the *Einleitung* Hegel is primarily concerned with *collective* subjects, philosophy and empirical science as agencies of knowledge-production, which need not exhibit the same structure. In Hegel's parlance, psychology belongs to finite spirit whereas philosophy belongs to infinite spirit. Accordingly, we should not assume that what Hegel says about the former process necessarily pertains to the latter. Instead, our interpretation should try to understand the conceptual transformation detailed in the *Einleitung* on its own terms. (Burbidge [2009], for example, mixes Hegel's two treatments of this transformative process and, I believe, consequently confuses the matter.) For recent work on Hegel's psychology, see Winfield (2007); Ziglioli (2016); Ikaheimo (2017); Moyar (2021).

in which thinking appears to itself as an object of consciousness or the kinds of "*content* that fills our consciousness" and constitutes thinking's "*determinacy*" (EL §3/GW 20:41). Because the difference between them lies in their form, the content of thinking "remains *one and the same*" across these forms (ibid.). This content-preservation among the forms of thinking explains why philosophy's conceptual transformation of empirical science cannot "simply set aside [its] empirical content" (EL §9A/GW 20:49).

As already noted, these forms of thinking are divided into three kinds for Hegel: representations, thoughts, and concepts (EL §3/GW 20:41f). *Vorstellungen*—perceptions, feelings, intuitions, images, ends, duties, desires, volitions—are the universals cognized immediately in ordinary sense-perception, corresponding to the generic sense of thinking. Representations are thus Hegel's way of denoting what we ordinarily mean by concepts, such as 'red', 'rose', and 'plant' (EL §24Z/W 8:83; EG §456Z/W 10:266). These terms are universals, but they involve no conscious reflection or 'processing' on the part of the subject who deploys them. Instead, in representational thinking the cognizer has "immediate consciousness of this content" (EL §6) and naturally uses representations to refer immediately to things in the world or their kinds and properties. But because these representations are implicitly universals, they have meaning only insofar as they implicitly refer to concepts. This is why our ordinary consciousness, according to Hegel, necessarily involves intermixing sensuous content with universal categories (EL §3A/GW 20:42f).

Gedanken are cognized in empirical science. Thoughts, or "finite thought-determinations" (EL §25/GW 20:68), distill representations into pure universals through a process of *analysis*, isolating from representations that content that no longer refers directly to sense-experience (EL §38Z/W 8:109). Examples of *Gedanken* include genera, species, laws, forces, matter, faculties, activities, and, most importantly, theories [Theorien] (EL §7/GW 20:46; EL §9/GW 20:49; EL §38/GW 20:75f; EL §80Z/W 8:169; GW 18:237). Two remarks can be made regarding thoughts (*Gedanken*) as a form of thinking (*Denken*).

First of all, we must note that, for Hegel, empirical science *already* involves itself with pure universals and does not rely upon philosophy to rise to this level of abstraction. This is why Hegel writes in our guiding passage that philosophy "recognizes and utilizes as its own content the universal produced by these sciences" (EL §9A/GW 20:49). For Hegel, this is because empirical-

scientific cognition includes the process of reflection that he calls *Nachdenken*.[26] *Nachdenken* differs from representational thinking because it is a form of thinking that takes thoughts explicitly as its subject-matter. As Hegel puts it, *Nachdenken* is "*reflective* thinking which has *thoughts* [Gedanken] as such for its *content* and brings them as such to consciousness (EL §2A/GW 20:40). *Nachdenken* has two features notable for my argument. First, Hegel characterizes it as a process of conceptual transformation, writing that in *Nachdenken* "the true *content* of our consciousness is *preserved* [erhalten] in its translation to the form of thought [Form des Gedankens] and the concept, and indeed only then placed in its proper light. […] *Nachdenken* has at least this effect, namely, that of transforming [verwandeln] the feelings, representations, etc. into *thoughts* [Gedanken]" (EL §5/GW 20:43). Second, *Nachdenken* is quite broad, encompassing, in a differentiated manner, both empirical science's analysis of the content of thinking and philosophy's conceptual exhibition of the same. Hegel writes that "[philosophy's] *Nachdenken* is both the *same* as and *different* from the first *Nachdenken* [viz., empirical science's] and, as such, it possesses in addition to the shared ones *its own peculiar forms*, of which the *concept* is the universal one" (EL §9/GW 20:49). The difference between empirical and philosophical science is that between a first and second *Nachdenken*, i.e., of a first- and second-order transformation in the form of thinking of one and the same content.

Second, these passages make it clear that we must carefully distinguish between *Denken* and *Gedanken* in the context of Hegel's philosophy of science, as I have done using thinking and thoughts, respectively. As we have seen, *Denken* refers to the generic conceptuality permeating all human experience. Representations, *Gedanken*, and concepts all exhibit, or are forms of, *Denken*. *Gedanken*, by contrast, refer not to the generic activity of thinking characteristic of humanity but to a specific object domain taken up in reflective mode, namely thinking about thoughts. *Nachdenken* is the activity that brings these reflective objects to consciousness; it can therefore be roughly understood as a species of the generic activity of *Denken*, the first of which produces *Gedanken* in empirical science and the second concepts in philosophical science.

This brings us at last to *Begriffe*, or true thought-determinations, cognized in philosophical science. Conceptual comprehension is well-known to Hegel's readers; this is what he sets out to

[26] On *Nachdeken* in the *Einleitung*, see Schick (2022, 87–88); Burbidge (2008, 35); Khurana (2011, 1140–41); Halbig (2002, 161–63).

achieve in his philosophical writings. I focus, then, only on two ways concepts are superior to thoughts, according to Hegel (EL §§7–10).[27]

First of all, concepts encompass a larger domain of objects than do thoughts, including within its sphere not only the finite objects of the sense-perception and empirical science but also *infinite* or "absolute" objects (EL §10/GW 20:50), i.e., objects that "in terms of their *content* […] immediately present themselves as infinite" (EL §8/GW 20:48). Hegel's examples of such infinite objects are freedom, spirit, and God (ibid.). One might expect these infinite objects to be philosophy's a priori contribution to knowledge. But Hegel immediately cuts off this misunderstanding. He writes that "the reason why they cannot be found in that sphere [of empirical cognition] is *not* that they are supposedly not part of experience" but because of their infinite content (ibid., emphasis added). This infinite content becomes a justified subject-matter of science by being part of philosophy's distinct conceptual transformation.

Second, philosophy gives its *Begriffe* the form of necessity, whereas empirical science's *Gedanken* are unsatisfactorily contingent in at least two ways (EL §9/GW 20:49). Regarding the first source of contingency, empirical science's universals are contingent because of its atomistic methodology (cf. Posch 2011, 189–92). According to Hegel, this atomism causes three deficiencies; he writes that, in empirical science, "the *universal* that it contains (such as the genus, etc.), is […] left indeterminate for itself and is not for itself connected to the *particular*; instead, both are external and contingent in relation to each other, as are likewise the combined particularities vis-à-vis each other in their reciprocal relationship" (ibid.). That is, empirical science's methodological atomism entails, first, that the particulars are left mutually external to one another—they are treated as isolated bits of data with no essential interconnection; second, that empirical sciences' laws and genera are left mutually external to the particulars over which they are intended to generalize—they are merely abstract universals, excluding the forms of particularity and singularity as such; and, third, that, as abstract universals, they fail to reach the level of explicitness demanded by conceptual comprehension and so are indeterminate. In short, empirical science's atomism prevents it from demonstrating that *these* laws and *these* genera are necessary features of the world's intelligibility, although it does prove that, *given the existence of certain laws*, relevant events in the world necessarily follow. The second source of contingency concerns the beginnings or *archai* of each empirical

[27] Hegel elsewhere identifies additional ways concepts are superior to *Gedanken* (e.g. EL §16A, §12A, §25). But these are inessential for my argument.

science. According to Hegel, empirical science cannot justify or prove these beginnings but must instead assume them as "*immediacies, found things, presuppositions*" (ibid.). Because it cannot demonstrate the necessity of its starting-points, empirical science remains vulnerable to skepticism about its soundness and therefore shows itself to be inadequate to the "form of necessity" proper to science (ibid.).

Philosophy's second *Nachdenken* resolves both of these deficiencies by way of self-development (EL §§10–12). That is, in philosophical science spirit develops out of itself the same content of empirical science, but now in the form of thinking as such or as "spirit [coming] *to itself*" (EL §11/GW 20:51). Philosophy's self-developmental method bestows necessity upon its system of concepts because, on the one hand, it eliminates the givenness of its starting-points by showing science to be a self-completing circle (EL §§15, 17; cf. SL 49/GW 21:57f), and because, on the other hand, it demonstrates the interconnectedness of its concepts by determining their content in the form of a self-unfolding system (EL §14/GW 20:55). This method also includes infinite objects within its domain ("such as the absolute, God") because, by turning to the form of thinking as such, it includes a moment of "*elevating*" itself above our "natural" or "sensuous" consciousness, negating this immediacy, and so making possible our comprehension of the "*universal* essence of these appearances" (EL §12/GW 20:52). In this way, philosophy preserves the content of the empirical sciences while transforming the form of its being thought from *Gedanken* to *Begriffe*.

In sum, in Hegel's philosophy of science philosophy is essentially a process of transforming the universals of the empirical sciences into concepts. I submit that this conceptual-transformative interpretation of philosophy makes the best sense of important claims in §§1–18, such as:

> It can be said quite generally that philosophy replaces representations with *thoughts* [Gedanken] and *categories*, but more specifically with *concepts*. (EL §3A/GW 20:42; see also EL §20A/GW 20:64)

> [O]nly by passing through the process of representing and by turning towards it, does thinking spirit progress to thinking cognition and to comprehension. (EL §1/GW 20:39)

> [T]he empirical sciences do not stand still with the perception of the *singularities* of appearances; instead, in thinking they have readied [gearbeitet] this material for philosophy by discovering its universal determinations, genera, and laws. In this way, they prepare

> [vorbereiten] this content of the particular so that it can be taken up [aufgenommen] into philosophy. (EL §12A/GW 20:54)

Philosophical science, including logic, is differentiated from empirical science not by being experience-independent but by the nature of its conceptual transformation—namely, this transformation's mode (necessity), cardinality (greater), and order (second).[28] This is what Hegel means when he writes that "the difference between them concerns solely this modification of the categories" (EL §9A/GW 20:49).

3.2 Towards a Cooperative View (EL §60A)

Nowhere does Hegel explicitly outline a successful process of scientific cooperation. Hegel does not show his work, as it were, and provides us only the systematic outcomes of this cooperative process. Its nature can thus only be reconstructed on the basis of the correctives that Hegel includes as *Anmerkungen* to the system. Two such moments in Hegel's logic are salient to my argument. The first is Hegel's critique of the infinitesimal in the *Science of Logic* operating on Hegel's cooperative model, treated in §4 below. The second is Hegel's insightful analysis in the *Encyclopaedia* logic of a breakdown in this cooperation when instead adopts a broadly Kantian view towards the philosophy-science relation, stemming from Gottfried Hermann's construal of the science of meter grounded in Kant's a priori (Hermann 1799). Hegel writes:

> This *further remark* may be added about the result concerning cognition, namely that the Kantian philosophy could not have had an influence [Einfluß] on the treatment [Behandlung] of the sciences. *It leaves the categories and the method of ordinary cognition completely unchallenged*. In scientific writings of the same, when they now and then start with sentences of the Kantian philosophy, the treatise shows in the sequel that those sentences were merely superfluous embellishment, and that the same empirical contents would have appeared, if those several initial pages had been dropped. (EL §60A/GW 20:98)

In a footnote, Hegel continues:

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[28] Given philosophy's second-orderedness, one can metaphorically say that philosophy operates at a further remove from sense-experience than empirical science. But this metaphor does not permit the further inference that Hegelian philosophy is "to a degree" a priori for the reasons outlined in note 11.

> Even in the *Handbook of Metres* by Hermann the beginning is made with paragraphs of the Kantian philosophy. Indeed, in §8 it is concluded that the law of rhythm must be (1) *objective*, (2) *formal*, (3) an *a priori determined* law. The reader ought to compare with these requirements and the subsequent principles of causality and reciprocity the treatment of the meters themselves, on which those formal principles have no influence [Einfluß] at all. (ibid.)

This apparently minor episode in the *Encyclopaedia* turns out to have significant ramifications for interpreting Hegel's views on scientific cooperation. If we negate this mistaken approach, we learn that philosophical science should "influence" the empirical sciences, where the nature of this influence is described by Hegel in two ways. First, this influence entails modifying the categories and methods of the empirical science to such an extent that it alters the empirical data relevant to this science on pain of formalism ("superfluous embellishment"). After philosophy's conceptual modification, certain phenomena become salient, others less so. Second, this influence arises at the level of principles: the philosophical part of a science transforms this science's principles, and these new principles should influence the empirical part's "treatment" of its subject-matter. Philosophy's inability to influence the practices of empirical scientists indicates that there is something deficient about it. Hermann's *Handbook*, then, combines philosophical and empirical science according to Hegel: the first, principle-determining part is philosophical, and the second, metre-treating part is empirical. This division is characteristic of all sciences, according to Hegel, corresponding to his view that each empirical science begins with a philosophical part responsible for laying out its "philosophical principles" or *archai* (EL §7A, §16). Hegel concludes that any philosophy capable of influencing empirical science cannot be a priori, as its resulting principles would be too formal to provide a guiding thread through the empirical-scientific material.

Departing momentarily from Hegel's vocabulary, we can accordingly generalize the view of scientific cooperation that follows from Hegel's example. Philosophy, when successful, influences the *research programs* of the empirical sciences, i.e., progressive series of scientific theories that unite particularized ways of explaining, choosing the relevant problems, methods, and data, and using one's basic concepts (Buchdahl 1993, 62; Lakatos 1970, 119, 132–38). Philosophy does so by providing empirical science justified and substantial non-formal principles and ameliorating the meaning of their basic categories, thereby preventing their misuse and its resulting confusion. However, this cooperation is not a one-shot procedure on Hegel's view. Instead, it arises from a

looping effect between philosophy's conceptual transformation and empirical science's theory construction: by "using" the results of empirical science (genera, laws, regularities, theories), philosophy in turn "influences" their research programs, leading these sciences to new results. These new results, in turn, require further conceptual modification through philosophy, transforming these new universals or *Gedanken* into concepts, and so on.

3.3 Empirical Revisability

At this juncture, one may question whether the empirical revisability of the cooperative view really distinguishes it from the insulationist and reconstructive ones. This is because the two alternative accounts of the philosophy-science relation can accommodate some form of revisability, at least at first blush. Moreover, depending on how this revisability is understood, it need not be incompatible with apriorism. But this indistinguishability is misleading; consequently, the incompatibility thesis is unthreatened. While it is true that certain varieties of the insulationist and reconstructive readings accommodate empirical revisability, they can do so only *externally*. On the cooperative view, by contrast, philosophical concepts are *intrinsically* empirically revisable, and it is this kind of revisability that is incompatible with apriorism. We can understand this difference by considering some examples.

For the insulationist view, take, for example, Houlgate's rendition of it. For Houlgate, both the philosophical concepts of nature and spirit, on the one hand, and of logic, on the other, are externally empirically revisable. Recall that on the insulationist view, the philosopher first unfolds the concept a priori and only afterward compares the deduced concept to the universals of empirical science. For Houlgate, such a method permits revision. The philosopher of nature, for example, may decide to revise his concepts in light of new discoveries by empirical science, as these discoveries "could alert the philosopher to problems in his articulation of the logic of nature" (Houlgate 2002, 117).[29] But the relation between philosophy and empirical science intended here by Houlgate is clearly external: new discoveries may indicate errors in our previous articulation, but empirical science bears no essential connection to this articulation as such. In principle, we could have

[29] Houlgate continues: "It is possible […] that further developments in empirical science could highlight hitherto unseen problems with Hegel's articulation of the logic of nature. […] Even if this were to occur, however, it would not make the revised systematic, logical structure of Hegel's philosophy of nature dependent on the deliverances of science. […] That logical structure would still be determined by the immanent, a priori logic of nature, now correctly articulated" (ibid.).

corrected these errors without the help of empirical science. Consequently, this revisability entails neither the dependence of *Naturphilosophie* on empirical science nor endangers the apriority of philosophy; it merely indicates that we have come, in an empirical way, to know eternal philosophical truths. Houlgate's view of logical concepts works similarly.[30]

Certain varieties of the reconstructive view can likewise accommodate empirical revision. Consider the non-metaphysical variety, e.g., Pinkard (1979, 417; 1981, 453f) and Hartmann (1966, 1972). According to this view, Hegelian philosophy proceeds first by examining the results of empirical science, taking these results as given inputs for philosophy, and then reconstructing these results in a priori form. It is therefore quite plausible to infer that if these results were to change in light of new empirical-scientific findings, then the reconstruction should be attempted again and thereby altered, but without threatening apriorism. As Hartmann (1966) acknowledges, "there may be a change in what we want to see granted in a reconstruction" (247).[31] Yet again, the intended relation between philosophy and empirical science is external: no intrinsic connection is intended between the empirical-scientific universals given to philosophy and its reconstructed conceptual scheme, and whether the latter exerts influence on the former is a matter of indifference to the reconstructing philosopher.

However, this external revisability faces two challenges as an adequate description of Hegel's philosophy of science. First of all, it apparently neutralizes the corrective function, motivating many proponents of the insulationist view, such as Stone, Longuenesse, and Bowman. To see this, consider an instance of conflict between philosophy's a priori conceptual scheme and a leading empirical-scientific theory, and, furthermore, suppose that no obvious error can be found on either side. On Houlgate's version of the insulationist view, such a conflict would indicate that philosophy's conceptual scheme, not the scientific theory, should be altered. Or, if one wishes to keep open the possibility that philosophy might correct the scientific theory, then a principled way of deciding these clashes would be needed. But I struggle to see what this principle could be, and the

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[30] See Houlgate (2006), wherein he distinguishes between "empirical" and "logical" concepts (which encompass all philosophical concepts, not merely those in the *Science of Logic*). Empirical concepts are "in principle," i.e., *intrinsically*, revisable, whereas logical concepts are only *externally* revisable (248, 251, 255). For Houlgate, it is the subject-matter, not the method or epistmeology, that changes in the transition from logic to nature.

[31] Hartmann (1972) later puts the point even more sharply: "Philosophy is historical in that it reconstructs a richness that is historical, and it is historical in that it is transient, provided that history changes so as to demand a new philosophy" (122).

clash seems to me undecidable; once we have put into doubt the certainty of a priori philosophy, what further standard remains? But providing this correction was among the primary functions of philosophy for the other proponents of the insulationist view. Something appears to go awry with revisable insulationists views like Houlgate's.[32]

Second, even if the trade-off between correction and revision can be successfully navigated, there remains a further issue for the insulationist and reconstructive views. This issue concerns the *externality* of the philosophy-science relation, namely, that such an external relation is incapable in principle of fostering the "influence" demanded in EL §60A. In that remark, Hegel clarifies that empirical science is not a mere "enabling condition," like the availability of a particular language with which I can communicate, oxygen to breathe, etc. (Winfield 2012, 33f; Houlgate 2006, 78f)—arguably its status on the reconstructive and insulationist views Rather, it is constitutive of the process of philosophy as such, and it is this *internal* connection between philosophy and empirical science that vitiates apriorism. Only by becoming conscious of its vocation to be efficacious in the process of knowledge—intervening in the conceptual field by expositing the universals of empirical science as a matter *internal* to philosophy—can it avoid being "mere embellishment" and actually alter the "empirical contents" of science.

The cooperative view, by contrast, satisfies the demand of EL §60A by positing an internal philosophy-science relation, thereby making its concepts intrinsically empirically revisable. On this view, philosophy begins, like the reconstructive view, by reviewing the best empirical-scientific theories within a particular domain of finite objects. But, on the one hand, it need not relate to these results as merely given, since they are taken to be influenced by previous philosophical-conceptual expositions. Philosophy thus recognizes itself as implicated in the universals of empirical science. On the other hand, philosophy relates to these results not *externally*, by denouncing, correcting, or accepting them, but *internally*, expositing and improving them.

Such an internal relation between philosophy and empirical science can neither be characterized as top-down, as on the insulationist view, nor as bottom-up, as on the reconstructive view, but as bilateral and cooperative. Both philosophy and empirical science require the other to undertake their distinct cognitive activities. Empirical science determines which kinds of things exist, theorizing about their nature; philosophy, in turn, ameliorates the meaning of its concepts, leading

[32] A similar impotency to challenge the results of empirical science afflicts the reconstructive view.

the former to further discovery and theorization. While practitioners of either kind of science may not be aware that theirs is a cooperative endeavor, and, indeed, may try to dominate the other party, the nature of their contribution to thinking remains essentially cooperative. Consequently, at least some of philosophy's concepts are empirically revisable in that they are *in principle* modifiable in light of new results in the empirical sciences. Philosophy even contributes to the necessity of its own revision, as its success leads these sciences to new discoveries in normal science and to paradigm shifts during periods of conceptual crisis. Finally, we reiterate that these consequences hold at the highest level of generality—they are features of Hegel's philosophy of science in §§1–18 and, as such, obtain for his entire system, applying univocally to logic, nature, and spirit.

4 The Infinitesimal

By way of conclusion, I turn all-too-briefly to an example of scientific cooperation in action in the *Science of Logic*, namely Hegel's critique of the infinitesimal in the calculus of his day (SL 204–70/GW 21:236–309). In doing so, I merely aim to exemplify the methodology underlying Hegel's views on scientific cooperation, showing it to be operative at a point in this text. For this reason, I focus exclusively on the methodological upshot of Hegel's critique of the infinitesimal, leaving aside the complex issues raised by Hegel's philosophy of mathematics.[33]

That being said, there remains one difficulty concerning Hegel's philosophy of mathematics that cannot be ignored if the example of his critique of the infinitesimal is not only to corroborate the cooperative view but also to provide evidence against apriorism. Namely, we must accept—counterintuitively—that Hegel understands mathematics to be an empirical science. While I cannot here defend this interpretation of Hegel's philosophy of mathematics in full, I will briefly indicate two reasons for thinking that this it is justified: first, that, mathematics' relation to philosophy is functionally equivalent to that between empirical science and philosophy, and, second, that mathematics deals with thoughts abstracted from experience in just the same way as other empirical sciences, such as physics, international relations, and political economy, deal with their basic theories and categories, and so is no less dependent on experience than these sciences are.

First, like all empirical sciences for Hegel, mathematics has its *Grundbegriffe* validated and explicated—in short, transformed—by philosophy. Mathematics can be called the science of

[33] For more substantial treatments of Hegel's critique of the infinitesimal, see Redding (2023, 132–39); Houlgate (2021, 2:209–44); Yeomans and Kaufmann (2017); Lacroix (2000); Pinkard (1981, 462ff).

quantity or, more precisely, the science of magnitude, where magnitude is understood as that which can be increased or diminished (SL 32/GW 21:37; SL 153/GW 21:175; SL 206/GW 21:239; EL §99A/GW 20:135; cf. PhG ¶45/GW 9:33). Its philosophical part (what Hegel calls "philosophical mathematics") concerns the meaning of our basic quantitative notions (*Größenbestimmungen*), such as quantum, number, ratio, degree, infinity, etc. This philosophical part of mathematics is treated within logic; in particular, the logic of quantity transforms and validates these mathematical universals, "deal[ing] with the concepts of [these] objects and generat[ing] their content through the development of the concept" (SL 205/GW 21:237). Within the structure of Hegel's philosophy of science, then, even pure mathematics, i.e., a formal and exact science, functions as an empirical science insofar as it follows the "method of the understanding": it lies between sense-experience and philosophy and purifies our representations of space and time into abstract *Gedanken* that attain hypothetical validity (i.e., validity conditions on accepting its axioms as given), but cannot prove the necessity of its object domain or the concepts with which it operates.[34] As such, mathematics, as with any science, must cooperate with philosophy if it is to attain genuine cognition. In the *Encyclopaedia*, Hegel puts this cooperative demand as follows:

> It is because mathematics is the science of the finite determinations of magnitude, which, in their finitude, are supposed to remain fixed and valid, and should not go beyond these determinations, that it is essentially a science of the understanding. […] It is always possible therefore that the concept may establish a more determinate consciousness, both of the guiding principles of the operations of arithmetic (cf. §102) and the theorems of geometry. (EN §259A/GW 20:250)

Indeed, if we follow Hegel's citation, we find in EL §102A a conceptual transformation of the basic arithmetic operations—addition, multiplication, raising to a power, and their respective negations—developed out of the concept of number itself (GW 20:137f). Mathematics thus functions as an empirical science for Hegel, working hand-in-hand with philosophy to produce genuine knowledge of the magnitudinal.

Second, insofar as Hegel understands mathematics as acquiring and justifying its universals by abstracting from experience, these universals remain to some degree *sinnlich*, just as in the case of

[34] Hegel asserts that mathematics deals with what can be "taken up lematically from experience" (SL 234/GW 21:272).

other empirical sciences, and this provides another reason for thinking that Hegel understands mathematics to be an empirical science. Hegel makes this experience-dependence most explicit in the case of the concept of number (*Zahl*). Number, Hegel tells us, is the "*thought* [Gedanke] of externality" which is "at the same time the abstraction from the sensuous manifold" and therefore "brings [the sensuous] closest to thought" (SL 178/GW 21:204). Number, then, is by no means pure but "stands between *the sensuous* and thought," just like the universals of the other empirical sciences (SL 179/GW 21:205).[35] As Hegel reportedly summarized in his lectures, "mathematics has to do with the abstractions of number and space; these are still something sensuous, albeit abstractly sensuous and without existence" (EL §19Z/W 8:70). Mathematical universals, then, are no more a priori than those of other empirical sciences, and thus Hegel's critique of the infinitesimal can be reasonably understood as exemplary of his views of scientific cooperation in a manner incompatible with apriorism.[36]

Let us now turn our attention to Hegel's critique of the infinitesimal proper. Hegel begins this critique in the *Science of Logic* with methodological remarks that should now be familiar. He praises the advances wrought by the infinitesimal—an infinitely small magnitude, represented by the *dy* over *dx* in Leibniz's notation, magnitudes which, as Hegel puts it, "*are in their vanishing*" (SL 79/GW 21:91; cf. Houlgate 2021, 2:222–25; Redding 2023, 92f)—but draws our attention to the "oddity that this science [of mathematics] has to date still been unable to justify its use of this concept," noting that, if mathematicians fail to take notice of (or ignore) philosophy's conceptual transformation of its basic notions such as the infinite, they will inevitably misuse them, generating confusion, even if mathematics continues to obtain "great results" based on these confused notions (SL 204/GW 21:236–37). The calculus is no exception. In utilizing the mathematical infinite without heeding its "critique" in the hands of philosophy, mathematicians "cannot determine the scope of its application and cannot secure itself against the misuse [Misbräuchen] of it" (ibid). One variety of this misuse and confusion is formalism, exemplified by Lagrange, whereby the calculus loses contact with "*that which is specific to the subject-matter*" (SL 260/GW 21:299). To continue making these

[35] In an *Encyclopedia Zusatz,* Hegel further asserts that mathematics shares the same methodological principle—that of the identity of the understanding—as two other empirical sciences, natural science and jurisprudence (EL §80Z/W 8:169f).

[36] It is no accident that Hegel complains that the "*a priori* is something altogether all too vague" in precisely his discussion of number (SL 173/GW 21:199).

advances and to comprehend them, mathematics and philosophy must work "hand in hand" (GW 24,1:490).

At the end of his remarks, Hegel summarizes his "critique" of the infinitesimal as follows:

> It has been the aim of these remarks to bring attention to the *affirmative* determinations that remain in the background, so to speak, in the various uses that are made of the infinitesimal in mathematics, and to extract them from the nebulosity in which they are shrouded when that category is held merely negatively. […] It is this [negative] determination that occasions the difficulty, a difficulty which can be resolved by an insight into its peculiarity and the simple nature of the subject-matter, but which, when the attempt is made to eliminate it by the aid of the infinite, only degenerates unresolved into confusion. (SL 269/GW 21:308f)

Hegel's theme here is clear: philosophy prevents mathematics from lapsing into confusion by ameliorating its notion of the infinitesimal. However, the nature of philosophy's intervention requires some elucidation. Striping away much of its complexity, we can reduce the issue to one of commensurability. According to Hegel, the calculus treats infinitesimals at times as finite magnitudes and, at others, as infinite magnitudes (SL 205/GW 21:237), which it does by abstracting from their qualitative differences (SL 603/GW 12:105). But, in treating one and the same thing in ways appropriate for two qualitatively different things (the arc as line, the continuous as discrete), it violates mathematical method, and mathematics is aware of this contradiction. This contradiction is the "negative" determination of the infinitesimal as qualitative incommensurability and alteration. The "affirmative" determination of the infinitesimal, by contrast, is the "qualitative determinateness of quantity" (SL 258/GW 21:297, 260/GW 21:299f), by which Hegel means the relation (ratio) that holds between the quantities in question. When taken in their relationality, the quantities shed their qualitative difference and become commensurable with one another (SL 270/GW 21:309).[37] This commensurability of the apparently incommensurable in the infinitesimal, overlooked by the calculus but implicit in its practice, underlies its success: "mathematics owes its most brilliant successes to precisely that determination which the understanding rejects" (SL 80/GW 21:92).

[37] As Redding (2023) puts it, "what has to be understood as continuous here is not magnitude per se but a continuous series of ratios between mangitudes" (94).

Three features of this cooperative episode are noteworthy for my argument. First, Hegel does not denounce the calculus's use of the infinitesimal but justifies it (cf. Houlgate 2021, 2:210). In fact, upon philosophical examination, the calculus bears even more legitimacy than its defenders suppose because the difficulty apparently posed by the infinitesimal's negative determination (its presumed leap from infinite series to finite quanity) finds its overcoming in its hidden affirmative determination. Contrary to its self-understanding, mathematics in the calculus does not violate its methodological principles. For this reason, Hegel's "critique" of the infinitesimal is significantly more constructive than what we ordinarily mean by that term.

Second, the character of philosophy's conceptual transformation is expository: it recovers what is implicit in the practice or "application" of the calculus (SL 204/GW 21:237; SL 223/GW 21:260; SL 227f/GW 21:265).[38] Using vocabulary common throughout the remarks, we may say that this exposition is possible because philosophical and ordinary mathematics share the same target: the nature of the matter (*Natur der Sache*) concerning the calculus. These sciences apprehend this *Sache* in different and complementary ways, such that it serves as the final standard of adequacy for both modes of cognition.

Finally, this influence is a two-way street for Hegel. Not only must mathematics heed philosophy to ameliorate its use of the infinitesimal, but the logic of quantity *intrinsically* depends on these advances in the calculus. On the one hand, Hegel publishes these remarks not only for the sake of philosophy or knowledge in itself but also to "influence" mathematics (recall §60A). This influence is practical in nature: mathematicians ought to read the *Science of Logic*, and after having learned the true meaning of the infinitesimal, subsequently change their respective research programs. If a philosophical science cannot in principle make this contribution to its empirical counterpart, as, for instance, is arguably the case with Kant's grounding of geometry and arithmetic in the a priori forms of intuition, space and time, then it is embellishment or, in a word, formalism (again, recall EL §60A). While one can question the success or even the feasibility of Hegel's wish—indeed, there is little evidence that Hegel's philosophy influenced the empirical sciences in the way

[38] Cf. Hegel's general remark in EL §88: "in general the whole progression in philosophizing […] is nothing other than merely the *positing* of what is already contained in a concept" (GW 20:12). Contra Houlgate (2021, 1:76, 2:245f), this passage, as I read it, does not support apriorism because part of what is contained in some concepts, i.e., their conceptual content, arises though experience and its analysis in empirical science.

that he had hoped, and certainly not in mathematics—there can be little doubt that he aimed at attaining this influence and saw it as part of the vocation of philosophy.

On the other hand, this downward influence still captures only one-sidedly Hegel's critique of the infinitesimal. For his part, Hegel only arrives at his insight into the calculus only through an internal process with ordinary mathematics, evident by his extensive engagement with the new literature on the topic. Such a process is not merely external. It is not only the case that if there were no mathematical science of the calculus—no Newton, Kepler, Carnot, or Lagrange—then Hegel would not have been able to developmentally unfold the logical idea to its fullest degree or in the same way—an external dependency that equally obtains for any enabling condition of philosophy—but that this science constitues an internal and formative moment of philosophy as such. Put positively, because ordinary mathematics functions as an empirical science in Hegel's sense, it produces "finding[s]" [Funde] (SL 284/GW 21:284) that cannot be foreseen by the philosopher but must be "taken up" [aufgenommen] into philosophy, even in logic (EL §12A/GW 20:54). In short, genuine knowledge of quantity requires cooperation between the philosophical and empirical sciences of the same. This holds just as much in logic as in nature and spirit. Moreover, this cooperation cannot be a priori because these empirical sciences of the understanding, even mathematics, are in part a posteriori, i.e., they contain empirical content and are justified in part by their reference to experience.

To conclude, in this article I have argued that, for Hegel, genuine knowledge arises only from the cooperation between philosophy and empirical science marked by an intellectual division of labor (the scientific cooperation thesis). In each case, the nature of this labor is a process of conceptual transformation: in the latter, from representations to *Gedanken*; in the former, from *Gedanken* to concepts. On the cooperative view I have proposed, philosophy essentially depends on empirical science to "prepare" [vorbereiten] its content, at least in part, and finds itself in a feedback loop with the latter, seeing the results of its conceptual transformation spur new findings, which in turn demand further philosophical comprehension. Because philosophy's conceptual transformation essentially involves empirical science, its concepts are intrinsically empirically revisable, and neither its content nor its justification is a priori (the incompatibility thesis).

Admittedly, our description of this cooperative process remains vague, demanding further explanation and exemplification in Hegel's texts. But my aim in this article has not been to decide the difficult matter of apriorism in Hegel conclusively, but to encourage further exploration of this

issue and the cooperation alternative to the insulationist and reconstructive views. Nonetheless, given the complex intertwining of philosophy and empirical science I have sketched in this article, we can begin to appreciate why Hegel abandons the notion of the a priori. Reimagining a scientifically cooperative Hegel not only lends support to broadly naturalist readings of his system but also gestures to a radically expository reframing of the task of philosophical critique. Critique, on Hegel's scientific-cooperative view, aims to exposit the insights, discoveries, and theories of the empirical sciences, furthering their ends by ameliorating their conceptual apparatus, not to debunk them.

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